Discrete Mathematics (SC205) Project

Assigned by:

Prof. Manish Gupta & Prof. Manoj Raut

Made by:

Suyash Bhagat(202101085)

Khushi Shah(202101430)

Vansh Joshi(202101445)

Keyur Rathva(202101465)

Sakshi Patadiya(202101469)

Ishita Rathod(202101516)

Akshar Panchani(202101522)

CONTENTS

Railway Planning and Delivery Systems:

Abstract:

In the railway sector, train scheduling has been a serious problem. Numerous methods and tools have been created in recent years to compute railway scheduling. In this research, we strive to schedule the trains so that they do not collide while traveling along the same path by presenting a set of heuristics for a constraint-based train scheduling tool. Train scheduling is formulated as a constraint optimization problem. By eliminating delays, we can shorten the passengers' trip time as well as improving the system. Additionally, it has been extremely difficult to plan delivery routes for postmen, salespeople, delivery men, or to plan the sequence of pick-ups for a school bus, etc. Solving such issues involves the highly regarded discrete mathematics discipline of graph theory. This issue calls for a circuit that travels via several different locations. Each location needs to be visited once. And the amount of time and fuel required should be kept minimum. In the realm of developing delivery routes, these issues are solved by employing graph theory and optimization.

I. Introduction:

A. *Railway Planning:*

The Ministry of Railways, Government of India, essentially owns Indian Railways. A National Rail Plan (NRP) for India upto 2030 has been created. The goal is to build a railroad system that is ”future ready” so that it can develop strategies based on both operational capabilities and commercial policy measures to enhance the railroads’ modal share of freight. There is a need to improve rail capacity because it frequently turns out that there aren’t enough trains to meet public demands. Occasionally, few trains carry just a small number of passengers. Therefore, careful train scheduling and planning are urgently needed to reduce loss, adhere to, and satisfy public demands.

B. ***Delivery System:***

Routes of the postmans, sales men, delivery guys, etc are planned using the graph theory. They have to travel to each of the places once and return to the place where they started. However, the time taken and distance covered must be minimum to make it effective in terms of cost and time.

So, the person starts from one place, goes to several places, each of them once and returns to the original place. To complete this in minimum time and/or traveled distance, it could be said that the person should pass from one path just once.

This is a combination of Eulerian circuit and Hamiltonian circuit as each edge as well as vertice needs to be traveled only once. Such a solution would be an optimal solution. However, it may not be possible to find such a circuit every time. So, in that case, one can find a circuit with repetition of minimum number of edges.

Graph theory is used to solve such problems. We have tried to explain how graph theory works and how it is used to solve these problems.

II. HISTORY:

A. *Railway System:*

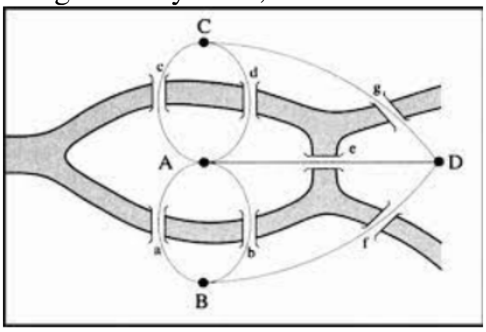
What requirements does a railroad system have? What does a railroad network constitute? What is visible is the infrastructure (also known as the ”fixed stock” of the network of railroad tracks, the stations, the yards, the signaling systems) as well as the rolling stock, which consists of the locomotives, carriages, and wagons that operates on the trains and transports people and product. The speed and energy efficiency of trains have significantly risen thanks to technical advancements, from the steam locomotives of the 19th century to the magnetic-levitation trains of today. The capacity, comfort, and convenience of rail travel have risen due to advancements in planning the system's operations, which are equally as significant as engineering changes and mathematical scheduling to the equipment.

*1) **Contributions from the past:** :* The train industry is actively conducting research in the field of timetable planning. Yang et al'sR research examines how incomplete cyclic train schedules are created. Wegele et al'sR study on real-time train scheduling uses genetic algorithms to heuristically rearrange trains when delays occur in the rail network. An important contribution to the solution of linear programming problems was made by George Bernard Dantzig. For instance, the airline sector is able to plan personnel and assign fleets because of Dantzig's expertise. Based on his research, tools that shipping corporations use to decide how many planes they need and where to deploy their delivery trucks are created.

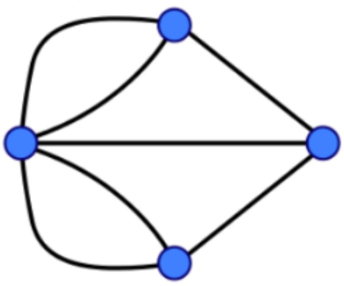
B. ***Delivery Systems:***

The origin of the delivery routing systems is traced back to a mathematical problem known as the seven bridges of Konigsberg.

The place has 7 bridges and four locations, the image of the same is shown below. Is it possible to leave from a specific location in Königsberg, take a walk that crosses each bridge exactly once, and then come back to the same location?



In 1736, it was solved by Leonhard Euler, a swiss mathematician. He provided a clever solution to the problem by representing it in the form of a graph where each of the places were described as a point and each of the bridges as the lines connecting them. It is shown in the figure below:



Euler suggested that the solution to this problem is that there is no solution to this problem. He said that it is not possible to find the solution to this problem.

He laid the foundation of graph theory. Graph theory is the basis for solving many problems, one of them being the delivery routing.

III. Mathematical Prerequisites:

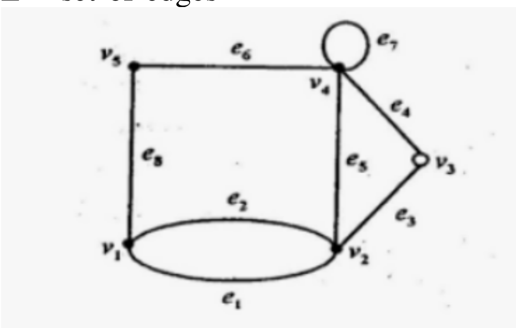
A. *Railway Planning and Delivery Systems:*

Discrete mathematics is used in the railway planning and delivery system to make decisions on how to expand train rail lines, schedule timetables, and schedule crews and equipment. A solid grasp of both linear algebra and graph theory is necessary for this.

*1) **Graph and types of graphs:*** : G = (V,E)

V = set of vertices

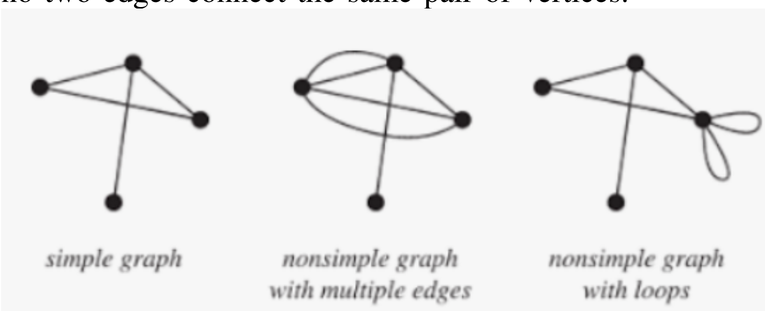
E = set of edges



V = (v1,v2,v3,v4,v5)

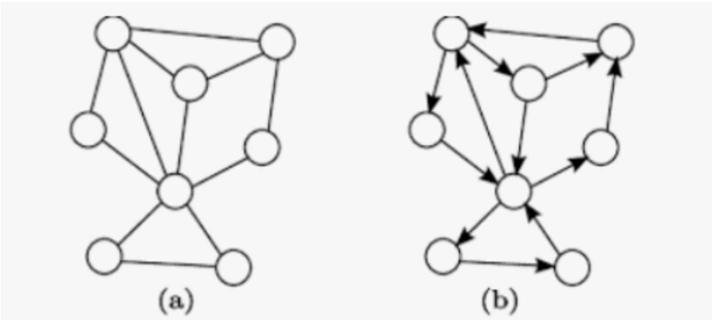
E = (e1,e2,e3,e4,e5,e6,e7,e8)

- **Simple Graph:** Each edge of the simple graph connects two different vertices and no two edges connect the same pair of vertices.



- Directed Graph: In a graph G = (V,E), an edge which is associated with an ordered pair of V*V is called a directed edge. The graph in which every edge is directed is called a directed graph.

- Undirected Graph: A graph in which every edge is undirected is called an undirected graph.



(a) Is an undirected graph and (b) is a directed graph.

2) ***Paths and Circuits**:* Path: A path in a graph is a succession of adjacent edges.

Circuit: A circuit is a path which joins a vertex to itself.

Simple Path: A simple path is a path in which no edge is repeated.

3) ***Degree:**:* - Degree of a vertex of an undirected graph is the number of edges incident with it.

Degree of a vertex of a directed graph:

Out-degree = Number of edges with the given vertex as their initial vertex

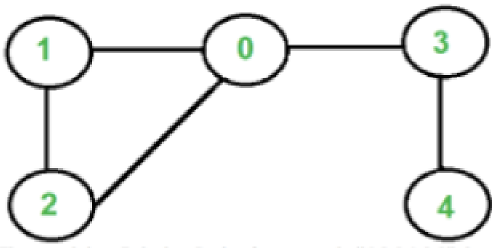
In-degree = Number of edges with the given vertex as their terminal vertex

4) ***Eulerian Path or Circuit**:* - Eulerian Path:

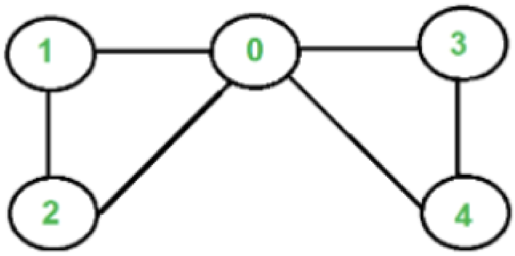
A path is said to be a Eulerian path if it traverses each edge in the graph only once.

- Eulerian Circuit:

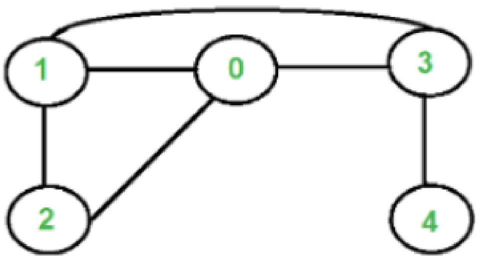
A circuit is said to be a Eulerian circuit if it traverses each edge in the graph only once. The difference between path and circuit is that for a circuit, it is necessary to end where it started but for path, it is not required.



The graph has Eulerian Paths, for example "4 3 0 1 2 0", but no Eulerian Cycle. Note that there are two vertices with odd degree (4 and 0)



The graph has Eulerian Cycles, for example "2 1 0 3 4 0 2" Note that all vertices have even degree



The graph is not Eulerian. Note that there are four vertices with odd degree (0, 1, 3 and 4)

5) ***Theorems to decide whether a graph can have a Eulerian Circuit or not :-** : -*

Theorem 1:

A connected multigraph has an Euler circuit if each of its vertices has even degree. **-**

Theorem 2:

A connected multigraph has an Euler path but not a circuit if it has exactly two odd degree vertices.

6) ***Hamiltonian Circuit:*** - Hamiltonian Path:

A Hamiltonian path passes through each of the vertices of the graph exactly once.

- Hamiltonian Circuit:

A Hamiltonian Circuit passes through each of the vertices of the graph exactly once, except for the vertex it started from.

7) ***Theorems to decide whether a graph can have a Hamiltonian Circuit or not :-** :*

- Divas's Theorem:

If a graph G has more than n≤3 vertices and every vertex has degree at least n/2 then G has a Hamiltonian circuit.

- Ore's Theorem:

If G is a simple graph with n vertices such that n≥3 and deg(u) + deg(v) for every pair of non adjacent vertices u and v, then G has a Hamiltonian circuit.

8) ***Optimization:**:* The meaning of optimization is the action of making the best or the most effective use of a situation or resource.

Mathematical Optimization is the selection of the best element, with regard to some criteria. Optimization involves minimizing or maximizing a function by choosing input values from the given set of inputs. It is the process of finding the best solution to some function given in a defined domain.

IV. Existing Problems and Solutions:

A. *Railway Planning:*

1) Key Stages: : These are the points that are taken into consideration while solving a problem:-

Network planning – determining the location and connectivities of tracks, stations and yards.

Line planning – determining the origin/destination termini and route of lines and their frequencies so as to provide sufficient capacity for the passenger trips to be served.

Timetabling – determining the despatch times of trains for each line; ensuring minimal waiting times and transit times for all passengers is particularly difficult.

Vehicle Scheduling – assignment of locomotives and coaches to trips, and

Crew Scheduling – rosters and duties for the drivers are determined, work rules and skills compatibility make the assignment of operators to trips a difficult problem. **Real-time traffic** – Post all the strategic planning, one has to take care of the realization of the schedule i.e. external influences will result in delayed trains. Minor irregularities in the execution of a schedule may imply severe disturbances etc.

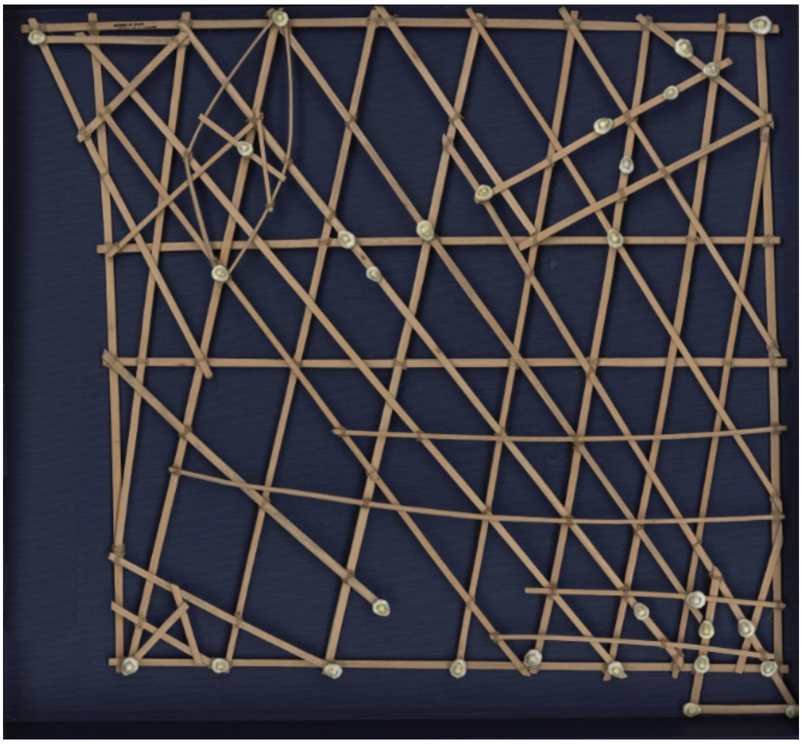
2) Existing Problems and Solutions:: Despite significant qualitative and quantitative advancements over the past several years, there are still a number of issues with the Indian Railways system that require immediate attention.

Although much has been managed, much more needs to be done. Following is a brief discussion of some of the significant issues the Indian Railways is now experiencing:

1. Railway Traffic Management: Numerous trains pass through crucial locations simultaneously. If disruption occurs, the traffic may be disturbed, which could lead to clashes. Multiple trains may simultaneously claim the same track section in a time of conflict. According to some reports, railways reported 20 major collisions on an average in a year. So, it is very important to manage traffic. As a result of this traffic, some trains need to be stopped or delayed to ensure safety because of which delays occur, which leads to several other problems such as travel-time management.

2. Minimization of passenger's travel-time and profit optimization: Some unexpected events may render some rail tracks partially or completely unavailable. So rescheduling is generally done. Usually in this situation, and also whether in normal scheduling, Rescheduling decisions are made exclusively from the perspective of the train, with the aim of preserving operation as much as feasible, minimizing train delays or minimizing deviations from the original schedule. But it is not beneficial for passengers. These delays not only affect the time table but also the passengers. So it is important to concentrate on passenger service and minimization of travel time.

In the past, in order to solve this issue, they used the Marshall Islands stick chart of Rebbelib type. This used to solve the problem of time scheduling and managing the trains.

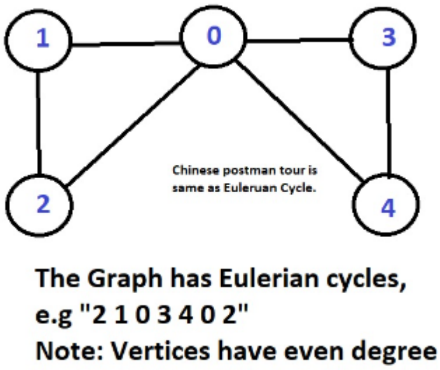


B. ***Delivery Systems:***

1) ***Chinese Postman Problem or Route Inspection Problem or Guan's Route Problem:***: This theory was originally stated by Kwan Mei-Ko and was later translated into English. These problems are aimed at finding the shortest path or circuit that visits every edge of an undirected graph. It is a Eulerian circuit in case the graph has one. But, if the graph does not have a Eulerian circuit then the solution to this problem is to find the circuit which traverses all the edges with the smallest number of edges repeated. We can easily understand that by the following examples.

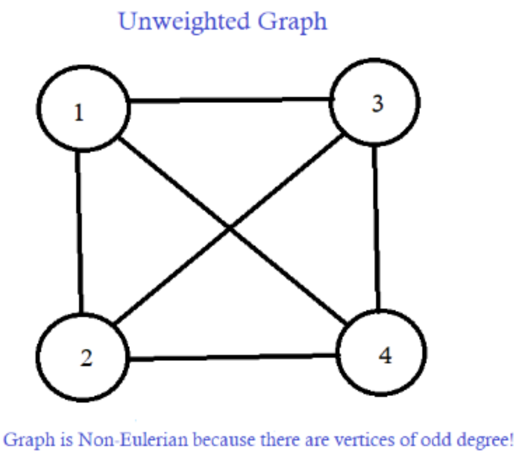
Examples:

1. Here, all the vertices of the graph are of even degree, hence the graph has a Eulerian circuit, so the optimized solution is a Eulerian circuit which is shown in the figure.



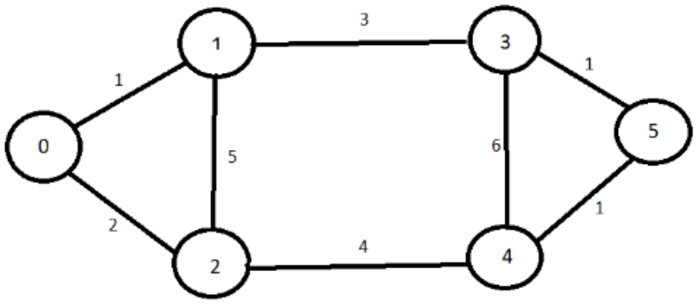
2. In the following example, the graph has four vertices all of them of odd degree. So, there is no Eulerian circuit possible for this graph. However, one can try to design a circuit in which a minimum number of edges are repeated.

Let's consider a circuit 1-2-4-3-1-4-3-2-1. Here, all the edges are traveled but the edges 1-2 and 3-4, each are repeated twice.



3. In this graph, the edges are weighted, so the weight must be minimum to find an optimized circuit in the given graph. One such solution is shown in the graph given below.

Weighted Graph



1-3-5-4-5-3-4-2-0-1-0-2-1

Chinese Postman Tour: Total Weight = 3+1+1+1+1+6+4+2+1+1+2+5

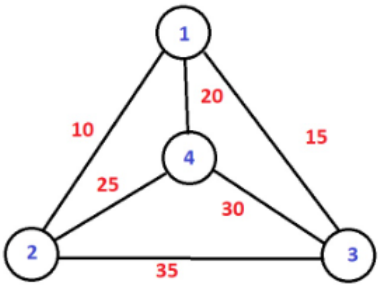
General solution to this problem is obtained by choosing T vertices which should be joined on a graph for odd degrees. This is called a T-join, this problem is solvable by polynomial time. A smallest T-join needs the necessary path which is half the mod of T joins. The time complexity of this problem in computational steps is O($n\exp 3$). For even numbers of odd vertices, in order to exist the T join we use a handshaking lemma to connect the graph.

2) ***Traveling Salesman Problem:***: This problem asks us the question: Given n points, find the shortest path which travels all n points exactly once. This same thing is used in DNA sequencing where we find a well formed approach to go through the DNA fragments and obtain an optimal solution. We should note the difference between Hamiltonian and Traveling salesman problems. The Hamiltonian cycle is to visit the point exactly once but we notice that there exist many such cycles, so the Traveling salesman problem uses the Hamiltonian cycle to find a solution with minimum weights. It tries to find the optimum

Hamiltonian circuit for the given graph.

Let's try to understand this by the following example:

Consider a traversal from 4-3-2-1-4. Here we notice the weights as 30+35+10+20 which equals 95. But a better way to find the solution is to traverse from 1-2-4-3-1. Which gives a weight of 80. Which is optimal in traversing the circuit. So, in such cases the optimal solution is the one with minimum weight.

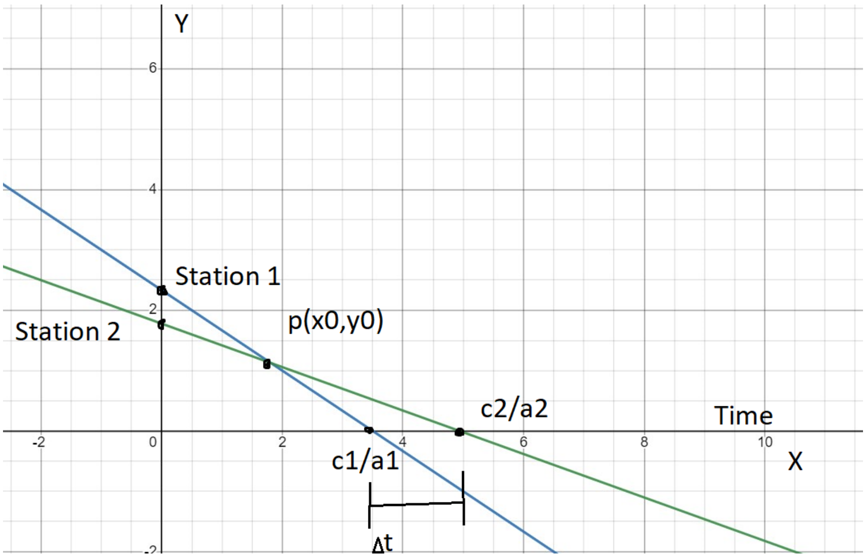


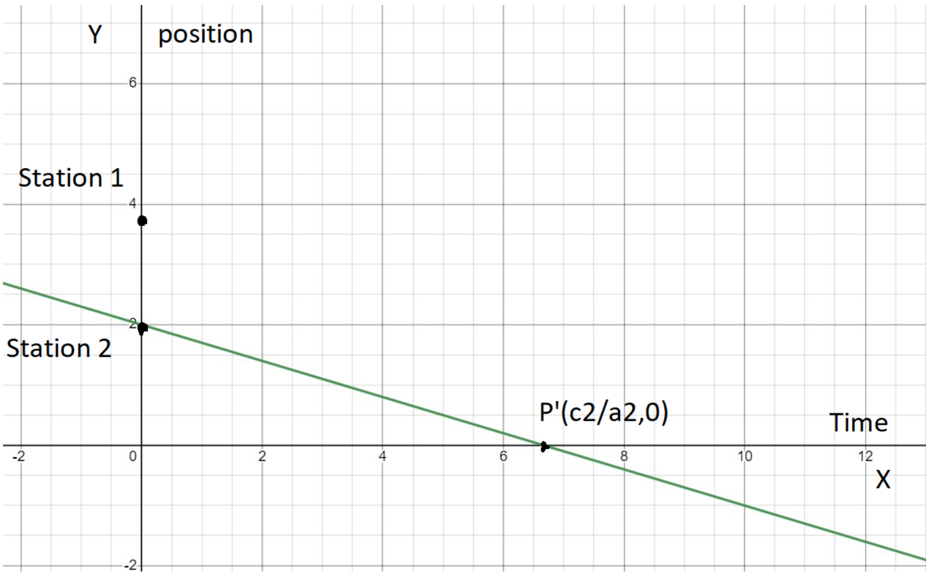
There are two major ways to approach this problem, which can be classified as the Permutation method and the other is the use of dynamic programming. Considering the method of permutation we start from one point and find all the (n-1)! Permutations for the obtained points, Now we would calculate the minimum cost and weight for the problem and will return the permutation with the Time complexity of $\Theta(n!)$.

V. OUR SOLUTIONS AND SUGGESTIONS:

A. *Railway Planning*

1. Editing the Departure Time for the equation





$t1 = a1x + b1y = c1$

$t2 = a2x + b2y = c2$ $P.O.T = P(x0, y0)$

The first Quadrant sends the travelling of 2 trains on the same track for particular amount of time.

for the negation position $\Rightarrow$ The tracks may change afterwards.

for the negation time $\Rightarrow$ There may be change of tracks of both trains and their different routes after which they are now travelling on same track.

Hence minimum delay of $\Delta t = \frac{c2}{a2} - \frac{c1}{a1}$ has to be done to avoid collisionof train t1.

Now equation of t2 $\Rightarrow$ $a2x + b2y = c2$

New equation of t1 $\Rightarrow$ delay of Δtx intercept

$\therefore x + \frac{b1y}{a1} = \frac{c1}{a1} + \Delta t$

$\frac{c1}{a1} + \frac{c2}{a2} - \frac{c1}{a1}$

$\therefore Speed = \mid slope \mid = \mid -\frac{a1}{b1} \mid$

$x + \frac{b1y}{a1} = \lim_{h \to 0+} (\frac{c2}{a2} + h)$

2. Edit the train speed

There are some constraints in train speed. The trains usually run at their optimum safe speed for fast and efficient travel.

Hence if a collision is probably happening between 2 trains on a particular path, then one of the train cannot increase speed as it will exceed the optimum safe speednand hence it must decrease its speed.

It is optimal to decrease the average speed and not continuously vary it by changing the acceleration as it would be more risky and less fuel efficient.

Also while decreasing the speed we need to ensure that there should be minimum delay in arrival of train at end of track.

Problem to minimise delay ; variable $\rightarrow$ speed

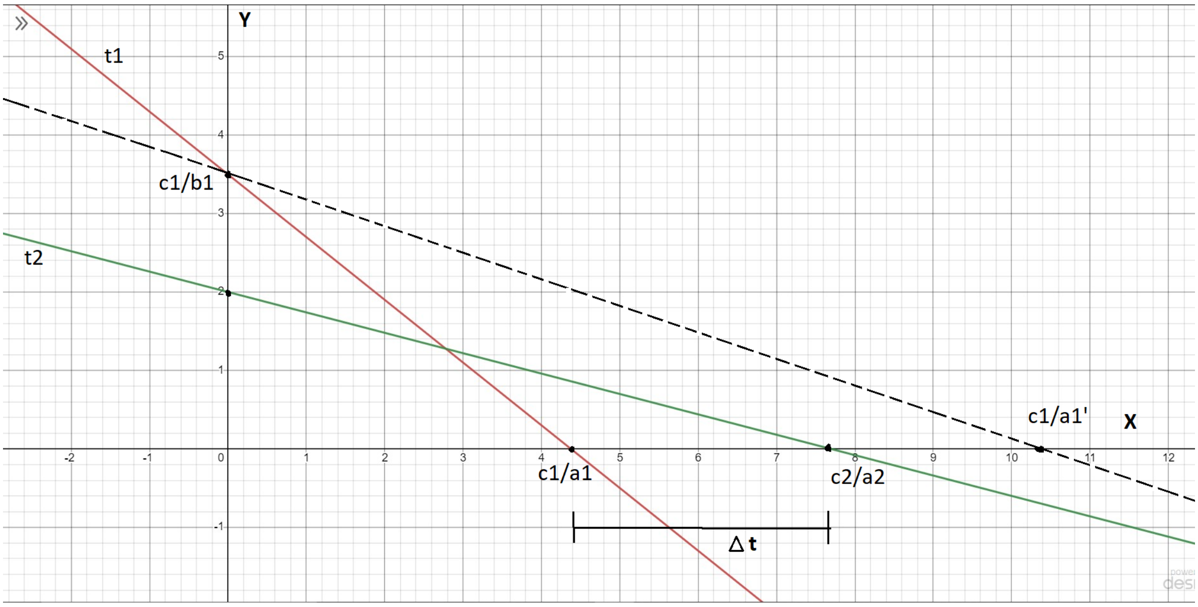
$t1 = a1x + b1y = c1$

$t1 = a2x + b2y = c2$

Hence speed $S1 =\mid -\frac{a1}{b1} \mid = \frac{a1}{b1}$

$S1 =\mid -\frac{a2}{b2} \mid = \frac{a2}{b2}$

$\Rightarrow t1 : s1x + y = \frac{c1}{b1}$, $s2x + y = \frac{c2}{b2}$



$\Delta t = \frac{c1}{a1'} - \frac{c1}{a1}$

$\Delta t = c1[\frac{1}{s1'b1} - \frac{1}{s1b1}]$

$\Delta t = \frac{c1}{b1}[\frac{s1-s1'}{s1s1'}]$ Condition minimum decreasing in speed $s1' = \frac{a1'}{b1}$ so to avoid collision

$\frac{c2}{a2} < \frac{c1}{a1'} < \infty$

$\therefore \frac{c2}{a2} - \frac{c1}{a1} < \Delta t < \infty$

To minimise it we get, $\Delta t = \lim_{h \to 0}(\frac{c2}{a2} - \frac{c1}{a1} + h)$

$\therefore$ Substituting values of Δt, we get $s1' = \lim_{h\to 0}(\frac{(s2b2)c1}{c2+hs2b2})$

$s1' \cong \frac{s2b2c1}{c2b1}$

Equation $t2$ = Same as before $\Rightarrow a2x + b2y = c2$

$s2x + y = \frac{c2}{b1}$

For $t1 \Rightarrow (\frac{s2b2c1}{c2b1})x + y = \frac{c1}{b1}$

B. *Delivery Systems*

The present and the early system of delivering products or goods was based on motion studies which includes factors such as, residential signature, business area, residential driver release. Also this all works on the basis of the timing of the order placed that is the part of the delivery sequence, let's consider the example if the delivery man is on a particular spot and the next location is nearby to him, he is supposed to go to the other location as given in his delivery schedule rather than to deliver the good at nearby point. This makes the process hectic and even tedious, even the monitoring of such data is not carried out in an efficient way.

To begin solving this issue, we must first do a thorough analysis of it. We require all delivery addresses as well as the deadline for the delivery of the item. Then, we must map each of these places and the routes that link them.

It is not always the case that minimum distance means minimum fuel consumption. We also need to take traffic into account. Other things such as traffic lights, the number of stops, city streets versus the highways etc. These things can be improvised with more upgrades .

We can cluster nearby stops on the delivery route. This is good as it avoids multiple trips to the same destination. For example, there could be some cluster of places where the vehicle can be parked at a place and the delivery person can travel around on foot to deliver the products. All customer's needs need to be met. Some deliveries are more urgent than others, they require higher priority. So, there are many problems involved here. The existing solutions try to take maximum of these problems into consideration.

Let's consider the location of the delivery points in the city to be the vertices of the graph and the paths connecting them to be the edges of the graph. So, the problem boils down to finding an optimum hamiltonian cycle in the given graph.

The most efficient circuit is a hamiltonian circuit with a minimum number of edges traveled. That means the person will travel all those delivery points exactly once before returning to the place from where he started. Also, he would be traveling a minimum distance and will cover a minimum number of edges. It would be an n-cycle if n locations need to be traveled. Using Eulerian method it takes time complexity of $O(|V|^2|E|)$.

It might not always be possible to get the most efficient solution which is to have a Hamiltonian circuit with minimum number of edges traversed.

The approach described in the Chinese postman problem does not provide an efficient solution to the problem we are trying to solve. It suggests the traversals of each of the edges at least once. But what we need here is the traversal of each vertex exactly once. So, the approach described in the Traveling Salesman Problem could be quite useful in designing the delivery routes.

So, our suggestion is to use the method described in the Traveling Salesman Problem to design a delivery route.

A detailed **Algorithm** is described here:

1.Start.

2.Gather the information regarding the exact locations of the places to be covered and the place from where the person starts his journey.

3.Map these locations and the connecting paths as a graph which has its vertices as the locations to be traveled and its edges as the connecting paths.

4.Now, mention the lengths of all the edges in the graph.

5.Start from the vertex from where the cycle begins.

6.Move to the nearest vertex other than the vertices that you have already visited. (there has to be a connecting path between them)

7.Repeat step 5 until you reach the starting vertex.

8.Stop.

The circuit made will yield a fuel and time efficient solution. A huge problem that we are facing is that there might not be paths connecting every pair of vertices of the graph. So, we may not find the most optimal solution to the problem sometimes. In that case if we have n locations to be traversed, if we make an n-cycle, that could be the most efficient solution as it would have n vertices and n edges. This n-cycle can most efficiently be made if we use drones for delivering goods. Even though it reduces manual work and faster delivery of goods, this can be used only when the package size and weight is manageable by the drones, and in all other cases, we will have to stick to the original solution of finding the most optimal Hamiltonian cycle.

C. *Thoughts on Commercialization*

A. Railway planning The Idea of commercialization doesn't fall into this category as the whole system of railway planning and it's commercialisation and manufacturing is worked by the Government , The Ministry for Railways, But the solution for the problem, the

scheduling the planning for rail routes, can be much more optimised by the above solution. This can help the rail department in improving the efficiency of their management , reduce the delays in the planning for the trains. It can reduce the time of traveling for passenger and for the manufacturer it reduces the traveling disctance ultimately reducing the fuel usage. So this idea can be used or can be given to the manufacturer and the rail departments, So this can be implemented in making its application or software also.
B. Delivery System. Delivery System and the networks is a vast field in which many public and private sectors are working. Many companies are formed using this method and idea, so the problem and the solution of the given project cannot be directly commercialized but it can be used by many delivering companies to increase their work efficiency and also decrease the delivery time. This notion can be adopted by the companies or the new emerging startup industry. We can design applications using this method and idea for the same.

VI. References

1. Chuang L. Elements of Discrete Mathematics. In: McGraw Hill Co-Singapore, Singapore: 1977, p-440.

2. Kenneth B. ,Clifford S. ,Robert L.. Discrete Mathematics For Computer Science. In: Key College publishing, Emeryville, CA : 2006 ,p-425.

3.Jin Z. , Wei W., Li M. .Task selection and scheduling for delivery. In: IEEE global conference, Abu Dhabi, UAE: 2018.

4.Yang D, Nie L, Tan Y, et al. Working on incomplete train timetable for fast speed rails . In:CA Brebbia , B Ning and N Tomii Computer in rail network XII. Southampton, UK: WIT Press, 2010, p.889

5.Corman F and Wegele S.. Comparison of the effectiveness of 2 real-time train scheduling system. In: I Hanson (ed.) Timetable plan and quality of information. Southampton, UK, Press, 2010, pp.189–198.

6.Lambert M. Glossary in theory of Graphs. In Route problem. by Betascript publisher in 2015,pp-90.

7.Anand N. and Anayi M., 2010.On Improving punctuality traffic in railway network in Sweden :through simulation. Rail Transportation Dept.,2021, Texas, USA,pp. 19.

8.Han B and Pie L.,Headway time in train timetable and its schedule. B.J University, Beijing , China. In 2015. pp-15

9.Bisen S.K. Graph theory in rail network and its transport. , J.G University,India. In 2017. pp-17.